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HIERARCHIAL COMMAND, CONTROL AND COUPLED BILINEAR SYSTEMS

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HIERARCHICAL COMMAND, CONTROL AND COUPLED BILINEAR SYSTEMS[1] (Invited 1984 IFAC World Congress, Proc.)

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Abstract. Models of a coupled bilinear structure are studied here. Included are applications to immunology and generalized defense processes. In some cases, coupled bilinear systems arise naturally and in others they represent valid approximations to more highly nonlinear systems. A basis is developed for multi-level command and control. Finally, hierarchical pursuit-evasion strategies are introduced, and plans are presented for more detailed studies.

Keywords. Bilinear control; cybernetics; game theory; hierarchical systems; and modelling.

INTRODUCTION

The purpose of this paper is to analyze the hierarchical structure of generalized defense and pursuit-evasion games which are amenable to bilinear or linear system analysis. Also, coupled bilinear systems, CBLS, sometimes arise in a natural manner for complex physical processes. In other cases, they may be developed to approximate input-output dynamics of a more highly nonlinear nature. Cellular fission, nuclear fission, convective and conductive heat transfer and certain chemical reactions are good examples of the former, while rigid-body dynamics, such as appear in aircraft, ship and robotic control, are good examples of the latter. Even for many of these cases, however, the CBLS representation may be exact by a redefinition, and an increase in the number, of state variables. BLS and CBLS models for some of these are studied by Mohler (1973).

Time and space limitations only permit a preliminary analysis of this complicated problem along with a systematic overview. However, a base is established for future research with particular application to generalized defense processes (human and national).

STRUCTURAL PROPERTIES

For this paper, BLS are given by the state equation,

$$\frac{dx}{dt} = Ax + \sum_{k=1}^{m} B_k u_k x + Cu, \qquad (1)$$

where $x \in R^n$, $u \in R^m$ with components $u_1, \ldots, u_m$, and A, C, B_k, $k = 1, \ldots, m$, are appropriate real constant matrices. Sometimes, it is convenient to define the output of the BLS (1) by

$$y = Dx, \qquad (2)$$

where $y \in R^p$, and D is an appropriate real constant matrix.

It is readily seen that addending the state vector x, with $x_{n+1} = 1$, allows (1) to be written in the homogeneous form (HBLS); i.e., (1) with zero C.

Then it is shown by Brockett (1973) that systems with state polynomials (products of states) and quotients of states may be decomposed into new HBLS. Consequently, it is noted by Tenney (1984) that systems, which involve transcendental functions of the state, may be decomposed into "polynomial systems" by merely defining new state variables as the functions themselves. This generalization of BLS makes them and certain of their associated methodologies amenable to a larger class of nonlinear systems which includes rigid-body dynamics such as for maneuvering aircraft, ships, land vehicles and robots. Also, predator-prey processes, combat games, immunology, electrical-power generation, and heating-ventilating and air-conditioning (HVAC) processes have similar representations. All of these systems are quite complex with hierarchical structures of appropriate orders.

Within each BLS itself there evolves a certain hierarchical structure which is associated with a "canonical" decomposition such as convenient for its Volterra representation (assuming its existence, of course). In this manner, BLS (1) and (2) may be generated by

$$y = Dx = D \sum_{i=1}^{\infty} x_i \approx D \sum_{i=1}^{N} x_i, \qquad (3)$$

where

$$\dot{x}_1 = Ax_1 + Cu,$$

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$$\dot{x}_2 = Ax_2 + \sum_{k=1}^{m} B_k u_k x_1 + Cu,$$

$$\vdots \qquad (4)$$

$$\dot{x}_\ell = Ax_\ell + \sum_{k=1}^{m} B_k u_k x_{\ell-1} + Cu,$$

$$\vdots$$

with $x(0) = 0$ for convenience.

It is readily seen [Bruni, et al (1974); Mohler (1973)] that x_i, $i = 1, \ldots,$ correspond to the terms in the Volterra series, and the corresponding kernels are generated as a "nesting" [Rao and Mohler (1975)] of linear-system impulse responses according to (4).

The Volterra series for BLS may sometimes be approximated by a finite number of terms (4), and, in the case of weakly BLS, may be exactly represented by a finite number, or decomposed into a finite number of linear systems with outputs multiplied together to form inputs to successive linear systems according to (4). Consequently, a physical system approximated (or given) by a finite hierarchical structure associated with (4) is conveniently analyzed by linear-system theory.

PHYSICAL PROCESSES

The hierarchical surveillance, decision and control structure which evolves for CBLS is demonstrated here by several relevant examples. Figure 1 indicates the natural model which is derived from the chemical law of mass action and from cellular fission for the immune system. See Mohler, Bruni and Gandolfi (1980), and Mohler (1982).

First, the human-defense system includes various mechanisms to detect and to destroy an alien which is similar in structure to that associated with national defense. These include the humoral system (generating antibody, Ab), the cell-mediated immune system (CMI, generating macrophages, M_ϕ, cytotoxic T cells, T_c, helper T cells, T_h, and suppressor T cells, T_s), and stages of complement.

The latter consists of nine cascaded BLS (i.e., enzyme-protein reactions) which successively amplify enzyme concentration until it "drills" the alien (e.g., tumor cell). Breaking down the alien permeability, fluid leaks in until the alien cell may burst apart.

Ab receptors on the surface of certain white-blood cells detect aliens of specific chemical structure, and generate appropriate defense mechanisms to best isolate and destroy the alien substance (e.g., bacteria, chemical contaminants, tumors, etc.). Certain mechanisms are ineffective against certain aliens. E.g., Ab-complexes (Ab, T_h and M_ϕ) may even shield a tumor while complement (triggered by a specific Ab) may be effective. The system involves chemically specific and nonspecific surveillance, communication, command and control (C^3). M_ϕ, T_c and Ab seem to be capable of alien destruction on their own, but the most effective defense involves an orchestrated combination of these processes.

Feedback control evolves according to alien concentration and chemical structure, and according to concentrations of Ab, T_h and T_s. The latter turn-on and turn-off the humoral process according to its need for Ab and/or memory cells to establish immunity and a future secondary response to infection.

Hierarchical control even occurs in the generation of Ab classes. E.g., one antibody is more effective early in an immune reaction while another is effective later. Hence, the well-known switch-over phenomenon for IgM to IgG is shown to be somewhat optimum by Perelson (1977). Anatomically, there is a hierarchical preference in the circulation of immune function. B and T cell precursors migrate to the lungs rapidly, but they also spend only a short duration there. On the other hand, a large number migrate to the spleen more slowly, and are trapped there for a much longer time period.

Many of the immune control mechanisms, which are associated with the nonlinear feedback block in Fig. 1, are not well understood. But the plant itself is pretty well defined by CBLS as shown in Fig. 1. Detailed mathematical models are presented by Mohler, Bruni and Gandolfi (1980), and Mohler (1982).

Just as the immune response evolves in a hierarchical fashion, so does the evolution of the human from the fertilized egg. A single cell is programmed genetically with the necessary prescription, and life evolves by a sequence of carefully controlled chemical reactions, cellular divisions and differentiations. Again, CBLS formulate the basis of a mathematical synthesis.

National defense takes on a hierarchical C^3 structure similar to that of body immune defense. Naval, air and land forces are coordinated with each also having subforces to deal with alien intrusion. Central command headquarters may allocate forces and various defense mechanisms according to certain criteria of performance.

At the final level, competition between defense forces, x_i and y_i, similar to biological species, may be modeled by CBLS in the general form

$$\frac{dx}{dt} = -A_x y + B_x(u_x)x + v_x,$$
$$\frac{dy}{dt} = -A_y x + B_y(u_y)y + v_y. \qquad (5)$$

Here x, y represent opposing forces of appropriate composition and dimension; v_x, v_y are appropriate vector force replenishments; A_x, A_y are appropriate matrix, opposing-attrition coefficients such as for "aimed fire"; $B_x(\cdot)$, $B_y(\cdot)$ are appropriate matrix, self-attrition and/or generation coefficients. u_x, u_y are appropriate cross controllers. For "area fire" of constant level per force; $B_x(\cdot)$, $B_y(\cdot)$ are diagonal matrices, such that the ith diagonal element is the ith control component. I.e., $u_{xi} = -a_{xi} - d_x^i y$. Here, d_x^i is an appropriate constant row vector, and a_{xi} is a self-attrition coefficient. u_{yi} is defined similarly. This CBLS is a generalized representation of the traditional Lanchester model. Dolansky (1964), and Wozencraft and Moose (1984).

Hierarchical CBLS appear in numerous engineering applications such as chemical processes, nuclear power generation and heating ventilating and air-conditioning (HVAC) systems, see Mohler (1972). In the latter, for example, bilinear heat-transfer models are prominent. Decisions are made to use solar storage, heat pumps and evaporative or refrigerative air conditioning according to changes in weather and various energy demands and costs. The latter involves stochastic processes as do most of those discussed above. For HVAC systems, random weather predictions are significant in the command and control process, and actual random weather

variations can affect the model. Similarly, observations of alien state is statistical in nature. The typical sonar or radar tracking in national defense and chemical tracking in immune defense are excellent examples. The synthesis of CBLS filters for this problem is studied by Halawani, Möhler and Kolodziej (1984), and is shown to offer better performance than the extended Kalman filter.

While stochastic games are significant for such applications, only the deterministic case is considered next, as a prelude to the even more difficult problem. Here competition may take place between defense forces and alien forces.

PURSUIT-EVASION GAMES

Several-pursuers, one-evader deterministic games are the subject of this section. The solution to the game requires the introduction of a strategic variable, then the game of kind is studied to determine possible cooperations between two pursuers. The optimal solution to a linear quadratic game and a simpler form of this solution are given, yielding the general solution to the N Pursuer - one evader game. A sensitivity study leads to a simple hierarchical structure which greatly reduces the amount of computation required.

Let the position (or more generally state), x_i, of an evader, E with control v, relative to pursuer, P_i with control u_i, be given by

$$\frac{dx_i}{dt} = f_i(x_i, u_i, v). \qquad (6)$$

Here, x_i, u_i, v and f_i have appropriate dimensions.

The capture occurs whenever one pursuer P_i gets E within its terminal manifold expressed by the quadratic equation at terminal time, t_f:

$$x_i^T(t_f)T_{xi}x_i(t_f) \le r_i^2,$$

where T_{xi} is a given matrix and r_i a scalar. The team of pursuers minimizes an appropriate performance index J, which E maximizes. This defines a single Hamiltonian for the team of pursuers. Therefore whenever a pursuer is added to the game, one equation is missing. The corresponding missing variable will be called the strategic variable z_i, for player P_i. This variable will be defined such that when $z_i = 0$, P_i is of no use to the solution of the game, and the larger z_i, the more effective P_i is.

The Game of Kind

Once the setting of the game is known, those pursuers that will be relevant must be determined. One way to proceed is to define for each pursuer the area where a copursuer must be located in order to allow (or not) cooperation. For each pair P_i, P_j the space is divided into the following six zones:

i) $z_1P_{jo}(\overline{P}_i/P_j{}^*)$. If P_{jo} belongs to this zone then P_i will not play any role in any game involving P_j, provided that P_j plays optimally. In this zone, $z_i = 0$.

ii) $z_2P_{jo}(\widetilde{P}_i/P_j{}^*)$. If P_{jo} belongs to this zone then P_i can play a role in a game involving P_j only if E does not play optimally or if the presence of other pursuers forces E to deviate from the (P_j, E) optimal strategy.

iii) $z_3P_{jo}(P_i/P_j{}^*)$. If P_{jo} belongs to this zone then P_i will play a role in the (P_i, P_j, E) game.

$z_1Uz_2Uz_3 = zP_{jo}(P_j \ge P_i)$ = the case in which the outcome of the (P_j, E) game is lower than for the (P_i, E) game.

iv) $z_4P_{jo}(P_j/P_i{}^*)$. If P_{jo} belongs to this zone then P_j will play a role in the (P_j, P_i, E) game.

v) $z_5P_{jo}(\widetilde{P}_j/P_i{}^*)$. If P_{jo} belongs to this zone then P_j can play a role in a game involving P_i only if E does not play optimally or if the presence of the pursuers forces E to deviate from the (P_i, E) optimal strategy.

vi) $z_6P_{jo}(\overline{P}_j/P_i{}^*)$. If P_{jo} belongs to this zone then P_j will not play any role in any game involving P_i, provided that P_i plays optimally. In this zone, $z_j = 0$.

$z_4Uz_5Uz_6 = zP_{jo}(P_i \ge P_j)$ = the case in which the outcome of the (P_i, E) game is lower than for the (P_j, E) game. The six zones cover the whole space. Here, P_i, $\overline{P}_i$ and $\widetilde{P}_i$ refer to three different classes of copursuers in the game.

As an example, the N-pursuers, 1-evader version of the wall-pursuit game presented by Isaacs (1965) could be renamed N cutters and fugitive railroad, where the evader of maximum speed 1 is constrained to the y axis. The minimum-time problem is considered, where the pursuers P_1 and P_2, controlling their constant-speed heading, have the following respective characteristics: the capture sets are circles of radii 1 and 0.5; the maximum speeds are 2 and 1.5, and $P_{10} = (-4, 4)$. See Fig. 2.

The zones are the same for every pursuer identical to P_j but are a function of P_{10}. They were derived from the solution to the 1 vs. 1 game and according to geometric considerations. The same kind of definitions can be made for 3 pursuers; e.g.,

$$z_1P_{ko}(\overline{P}_k/P_i{}^*, P_j{}^*)$$

etc., but the derivation of the zones would require that the 2 vs. 1 game be solved first.

The Game of Degree

The game studied is a linear, quadratic, generalized 2-pursuer, 1-evader game with

$$\frac{dx_i}{dt} = A_ix_i + B_iu_i + C_iv, \qquad (7)$$

where

$$x_i(t_f)^T T_{xi}x_i(t_f) = r_i^2, \text{ and}$$

$$J = \frac{1}{2}\int_{t_o}^{t_f} \left(\sum_i x_i^TQ_ix_i + \sum_i u_i^TR_iu_i - v^TSv\right)dt \qquad (8)$$

is to be minimized by the pursuers and maximized by the evader.

1-pursuer vs. 1-evader game. Let λ_{pi}, λ_{ei}

be the costate vectors required by the game, such that the transversality condition is given by

$$\lambda_{pi}(t_f) = z_i T_{xi} x_i(t_f), \quad \lambda_{ei}(t_f) = z_i' T_{xi} x_i(t_f)$$

where z_i, z_i' can be any nonzero positive scalars. The Hamiltonians are

$$\begin{aligned} H_p &= (A_i x_i + B_i u_i + C_i v)^T \lambda_{pi} \\ &\quad - \frac{1}{2}(x_i^T Q_i x_i + u_i^T R_i u_i - v^T S v), \\ H_e &= (A_i x_i + B_i u_i + C_i v)^T \lambda_{ei} \\ &\quad + \frac{1}{2}(x_i^T Q_i x_i + u_i^T R_i u_i - v^T S v). \end{aligned} \tag{9}$$

The open-loop Nash solution to the game is

$$\begin{aligned} u_i^* &= R_i^{-1} B_i^T T_{pi} \Phi_i(t, t_o) x_{io}, \\ v^* &= S^{-1} C_i^T T_{ei} \Phi_i(t, t_o) x_{io}, \\ \dot{\Phi}_i(t, t_o) &= (A_i + N_i T_{pi} + M_i T_{ei}) \Phi_i(t, t_o), \end{aligned} \tag{10}$$

where $\Phi_i(t,t) = I$

$$N_i = B_i R_i^{-1} B_i^T, \quad M_i = C_i S_i^{-1} C_i^T, \text{ and}$$

$$\begin{aligned} \dot{T}_{pi} &= -A_i^T T_{pi} - T_{pi} A_i - T_{pi} N_i T_{pi} - T_{pi} M_i T_{ei} + Q_i, \\ \dot{T}_{ei} &= -A_i^T T_{ei} - T_{ei} A_i - T_{ei} N_i T_{pi} - T_{ei} M_i T_{ei} - Q_i, \end{aligned} \tag{11}$$

where

$$T_{pi}(t_f) = z_i T_{xi}, \quad T_{ei}(t_f) = z_i' T x_i.$$

To this solution, another two conditions, restricting the solution must be added. I.e., $z_i \geq 0$ when substituted into the Hamiltonian provides the playability condition, and

$$\lambda_{pi}^T(t_f) \dot{x}_i(t_f) \leq 0$$

is the capture condition.

The solution is a classical one. A complete derivation of the two-player, linear, quadratic game is given by Ichikawa (1976) and Hamalainen (1978). A procedure to decouple the Riccati equations (11) is given by Simaan and Cruz (1973) taking advantage of a preliminary solution common to several initial conditions, reducing the problem to the computation of successive linear equations.

<u>2-pursuer vs. 1-evader game</u>. The four costate vectors required are: λ_{pi}, λ_{ei}, $i = 1, 2$, where

$$\lambda_{pi}(t_f) = z_i T_{xi} x_i(t_f), \quad \lambda_{ei}(t_f) = z_i' T_{xi} x_i(t_f)$$

The open-loop Nash solution to the game is

$$u_i^* = R_i^{-1} B_i^T T_{pi} \Phi_i(t, t_o) x_{i0}, \qquad i = 1, 2. \tag{12}$$

$$\begin{aligned} v^* &= S^{-1}(C_1^T T_{e1} \Phi_1(t, t_o) x_{10} + C_2^T T_{e2}\ {}_2(t, t_o) x_{20}), \\ \dot{\Phi}_1(t, t_o) &= (A_1 + N_1 T_{p1} + M_1 T_{e1}) \Phi_1(t, t_o) \\ &\quad + C_1 S^{-1} C_2^T T_{e2} \Phi_2(t, t_o) \\ \dot{\Phi}_2(t, t_o) &= (A_2 + N_2 T_{p2} + M_2 T_{e2}) \Phi_2(t, t_o) \\ &\quad + C_2 S^{-1} C_1^T T_{e1} (\Phi_1(t, t_o) \end{aligned} \tag{13}$$

where

$$\Phi_1(t,t) = \Phi_2(t,t) = I, \text{ and}$$

$$\begin{aligned} \dot{T}_{p1} x_1 &= [-T_{p1} A_1 - A_1^T T_{p1} - T_{p1} N_1 T_{p1} - T_{p1} M_1 T_{e1} \\ &\quad + Q_1] x_1 - T_{p1} C_1 S^{-1} C_2^T T_{e2} x_2, \\ \dot{T}_{e1} x_1 &= [-T_{e1} A_1 - A_1^T T_{e1} - T_{e1} N_1 T_{p1} - T_{e1} M_1 T_{e1} \\ &\quad - Q_1] x_1 - T_{e1} C_1 S^{-1} C_2^T T_{e2} x_2, \\ \dot{T}_{p2} x_2 &= [-T_{p2} A_2 - A_2^T T_{p2} - T_{p2} N_2 T_{p2} - T_{p2} M_2 T_{e2} \\ &\quad + Q_2] x_2 - T_{p2} C_2 S^{-1} C_1^T T_{e1} x_1, \\ \dot{T}_{e2} x_2 &= [-T_{e2} A_2 - A_2^T T_{e2} - T_{e2} N_2 T_{p2} - T_{e2} M_2 T_{e2} \\ &\quad - Q_2] x_2 - T_{e2} C_2 S^{-1} C_1^T T_{e1} x_1, \end{aligned} \tag{14}$$

where

$$T_{pi}(t_f) = z_i T_{xi}, \ T_{ei}(t_f) = z_i' T_{xi}, \quad i = 1, 2.$$

The solution is very complex compared to the 1 vs. 1 game. The differential systems, (13) and (14) are tightly coupled requiring parallel computation whereas the solution to the 1 vs. 1 game could proceed in several steps.

Note that only one of the two pairs of strategic variables (z_1', z_1), (z_2', z_2) can be fixed, the other one must be selected properly to correspond to the initial condition of the game.

Keeping the same controls, but setting $z_2 = 0$ $z_2' = 0$ and exploring all the possible $x_2(t_f)$ will give the help zone, $H = z_1 \cup z_2 \cup z_3 \cup z_4$, the area outside of which P_2 does not play a role in the optimal (P_1, P_2, E) game.

<u>Simplifying the 2 vs. 1 solution</u>, define Z_i such that

$$Z_i T_{e1} x_1 = T_{ei} x_i, \qquad i = 1, 2.$$

(Note that $Z_1 = I$, the identity.) Now, the solution, while (12) and (13) remain identical, is as follows.

$$\begin{aligned} \dot{T}_{p1} &= -T_{p1} A_1 - A_1^T T_{p1} - T_{p1} N_1 T_{p1} - T_{p1} M_1 T_{e1} \\ &\quad - T_{p1} C_1 S^{-1} C_2^T Z_2 T_{e1} + Q_1, \\ \dot{T}_{e1} &= -T_{e1} A_1 - A_1^T T_{e1} - T_{e1} N_1 T_{p1} - T_{e1} M_1 T_{e1} \\ &\quad - T_{e1} C_1 S^{-1} C_2^T Z_2 T_{e1} - Q_1, \end{aligned} \tag{15}$$

where

$$T_{p1}(t_f) = z_1 T_{x1},\ T_{e1}(t_f) = z_1' T_{x1},$$

$$\dot{T}_{p2} = -T_{p2}A_2 - A_2^T T_{p2} - T_{p2}N_2T_{p2} - T_{p2}M_2T_{e2} - T_{p2}C_2S^{-1}C_1^T Z_2^{-1} T_{e2} + Q_2$$

$$\dot{T}_{e2} = -T_{e2}A_2 - A_2^T T_{e2} - T_{e2}N_2T_{p2} - T_{e2}M_2T_{e2} - T_{e2}C_2S^{-1}C_1^T Z_2^{-1} T_{e2} - Q_2 \qquad (16)$$

where

$$T_{p2}(t_f) = z_2 T_{x2},\ T_{e2}(t_f) = z_2' T_{x2};\ \text{and}$$

$$\dot{Z}_2 = Z_2(A_1^T + Q_1T_{e1}^{-1}) - (A_2^T + Q_2T_{e2}^{-1})Z_2,$$

$$\dot{Z}_2^{-1} = Z_2^{-1}(A_2^T + Q_2T_{e2}^{-1}) - (A_1^T + Q_1T_{e1}^{-1})Z_2^{-1}, \qquad (17)$$

$$Z_2(t_f)T_{x1}X_1(t_f) = \frac{z_2}{z_1} T_{x2}X_2(t_f),\ Z_2^{-1}(t_f) = [Z_2(t_f)]^{-1}.$$

The similarities with the 1 vs. 1 game are now obvious; the equations are decoupled and can be solved in turn. However the problem of guessing the strategic variable remains. Note the simplification that $Q_1 = Q_2 = 0$ would introduce.

Consider Fig. 3 as an example of a 2-dimensional, linear, quadratic game with identical pursuers. Here, $Q = 0$, $R = S = -I$,

$$A = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix},\quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Also, u_1, u_2, v are scalars, $T_x = I$, $r = 1$.

The capture zone for the 1 vs. 1 game, defined by $J \geq -10^{10}$ is very limited, as is the usable part of the terminal manifold. However, for the 2 vs. 1 game, there is no limitation on the usable part, and the capture zone is much larger. Ending on identical points, variations on the strategic variable z_2 leads to different trajectories. The proper values for $x_1(t_f)$, $x_2(t_f)$ and z_2 must be selected in order to fit with the initial conditions x_{10}, x_{20}.

<u>Solution to an N-identical-pursuer, 1-evader linear quadratic game</u>. The solution, for $Q = 0$, takes the following form with $x_i(t)$, $x_i(t_o) = x_{io}$, given by (7):

$$u_i^* = B^T T_i x_i,\quad v^* = C \sum_i^{1,n} T_i x_i, \qquad (18)$$

$$\dot{T}_i = -T_iA - A^T T_i - T_i(M+N)T_i - T_iM \sum_{j\neq i}^{1,n} Z_j Z_i^{-1} T_i, \qquad (19)$$

$$T_i(t_f) = z_i T_{xi};$$

where

$$\dot{Z}_i = Z_iA^T - A^T Z_i,\ \dot{Z}_i^{-1} = Z_i^{-1}A^T - A^T Z_i^{-1},$$

with $Z_1 = I$,

$$Z_i(t_f)T_{x1}X_1(t_f) = \frac{z_i}{z_1} T_{xi}X_i,\quad Z_i^{-1}(t_f) = [Z_i(t_f)]^{-1}. \qquad (20)$$

Let
$$\beta_i = -T_iM \sum_{j=i}^{1,n} Z_j Z_i^{-1} T_i = -T_iM \sum_j^{1,i-1} Z_j Z_i^{-1} T_i + W_i$$

and
$$W_i = -T_iM \sum_j^{i+1,n} Z_j Z_i^{-1} T_i$$

Except for the addition of β_i and the sum in v^*, the solution is identical to the 1 vs. 1 game.

This suggests a sensitivity study on both β_i and W_i. In some instances the β_i are negligible when the pursuers represent equal threats to the evader. In this case the difference between the 1 vs. 1 game and the n vs. 1 game is seen to come hardly from the controls adopted by the pursuers but from the control adopted by the evader. β_i is an important term if P_i plays a minor role in the capture. On the other hand W_i is always negligible when the playability condition is not violated, provided that the pursuers be classified in order of importance, P_1 being the most important (the closest one to E for the minimum time problems). Consequently, the structure is shown in Fig. 4 where each pursuer solves (18) - (20), finding the couple $x_i(t_f)$, z_i corresponding to x_{io}, and passes this information forward.

A parallel structure can be derived from this "ripple structure," enhancing the independence of the individual pursuers with respect to the team, but at the expense of an increased number of equations to solve. A "minor" pursuer can be added but the gain produced by this pursuer must be weighted against the amount of delay or computation that this very pursuer will have to cope with. This structure, implemented for the previous example, did not show any variation in state, control or performance superior to 0.1% with the rigorous solution.

CONCLUSIONS AND DIRECTIONS

Convenient hierarchical structures for pursuit-evasion games (or command and control) are generated along with an overview of the role of BLS in generalized defense and other relevant applications. Future plans call for an integration of this analysis with stochastic nonlinear observation, and stochastic BLS models such as are relevant to estimation and control in the applications presented above.

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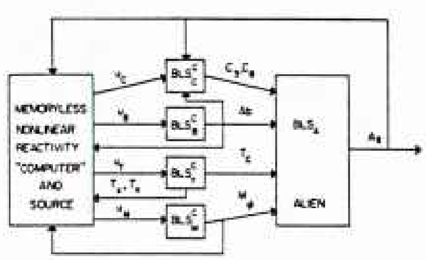


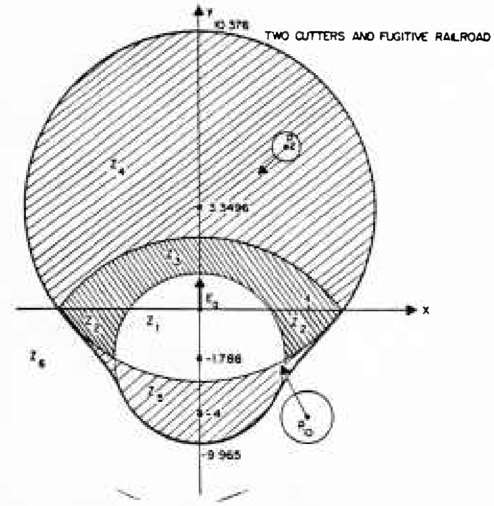
u refers to control reactivity with subscripts c for complement, B for B cells, T for T cells, M for macrophages.

BLS^c refers to coupled BLS.

C_3, C_8 refers to appropriate complement stages.

Other terms defined in text.

Fig. 1 BLS Synthesis of Significant Immune Processes



If: $P_{20} \in Z_1$: P_2 alone will capture E.

$P_{20} \in Z_2$: P_2 alone will capture an optimal E.

$P_{20} \in Z_3$: P_2 will be helped by P_1 in capturing E.

$P_{20} \in Z_4$: P_1 will be helped by P_2 in capturing E.

$P_{20} \in Z_5$: P_1 alone will capture an optimal E.

$P_{20} \in Z_6$: P_1 alone will capture E.

Fig. 2 Influence of the Position of P_2 on the Resulting Game. $P_{10} = (4, -4)$.

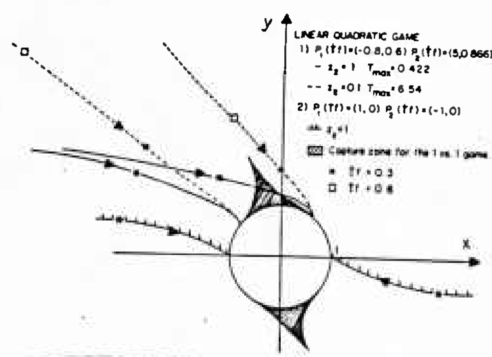


Fig. 3 2-Pursuer, 1-Evader Relative Trajectories Leading to Capture.

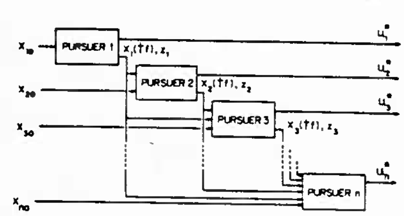


Fig. 4 n-Pursuer Suboptimal Hierarchical Structure.